Electronic Supplementary Material
Parameter estimation in a generalized discrete-time model of density dependence

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Figure S1 shows several different *pgr* curves for arbitrary r and different values of γ. We selected of r_0 values from the set {1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5} and γ_0 values from the set {1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5} (Fig. S2), keeping $K_0 = (r_0 - 1)^{1/\gamma_0}$ to evaluate the effect on parameter estimation for qualitatively different *pgr* curves.

We present parameter estimation statistics for samples from the stationary distribution with $\sigma_0 = 0.05$ and sizes $T = 10$, 20, and 50 in tables S1-S18. Figure S2 shows the location of the (r_0, γ_0) pairs in relation to the stability boundary $r_0 = 2/(\gamma_0 - 2)$ for the deterministic trajectories of the stochastic generalized Beverton-Holt model with $\sigma_0 = 0$. Table S19 gives these statistics from simulations with increased noise ($\sigma_0 = 0.25$), stationary and perturbed samples for the two representative (r_0, γ_0) pairs. Tables S20 and S21 are the variance-covariance matrices of the ML parameters at $\hat{\theta}$ for the two example time series described in the main text.

Table S1- $\hat{r}$ bias, T=10

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	3.30E+00	4.15E+00	5.36E+00	6.92E+00	7.51E+00	9.28E+00	1.60E+01	1.03E+01	9.23E+00
2	3.71E+00	6.50E+00	8.68E+00	1.85E+01	2.82E+01	3.49E+01	2.11E+01	1.03E+00	9.48E-02
2.5	4.61E+00	6.61E+00	1.32E+01	3.79E+01	6.33E+01	3.75E+01	2.71E-01	4.52E-02	4.04E-02
3	3.94E+00	7.38E+00	2.79E+01	4.99E+01	1.15E+02	8.05E+00	6.19E-02	2.69E-02	1.97E-02
3.5	4.26E+00	9.17E+00	3.49E+01	8.05E+01	2.33E+01	1.20E-01	2.37E-01	1.42E-02	1.06E-02
4	8.34E+00	1.10E+01	4.58E+01	1.38E+02	8.09E-01	1.12E-01	2.57E-02	4.30E-03	1.76E-02
4.5	3.59E+00	1.02E+01	4.36E+01	1.71E+02	1.16E+00	8.11E-02	1.48E-02	1.17E-02	4.88E-03
5	3.72E+00	1.34E+01	5.20E+01	2.24E+02	5.79E-01	7.11E-02	1.10E-02	6.69E-03	1.82E-02

Table S2- $\hat{r}$ variance, *T=10*

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	3.52E+01	5.67E+01	8.88E+01	2.89E+02	1.84E+02	3.75E+02	4.94E+03	4.83E+02	4.38E+02
2	6.42E+01	1.95E+02	2.72E+02	7.56E+03	1.84E+04	2.83E+04	1.38E+04	7.59E+01	1.40E+00
2.5	1.67E+02	1.66E+02	2.84E+03	3.29E+04	8.22E+04	6.52E+04	6.95E+00	4.60E-02	1.26E-01
3	9.56E+01	2.43E+02	1.60E+04	5.68E+04	1.95E+05	1.86E+04	4.20E-02	2.63E-01	1.51E-02
3.5	1.51E+02	6.85E+02	2.67E+04	9.79E+04	2.31E+04	1.98E-01	1.20E+01	3.16E-02	1.08E-02
4	6.14E+03	1.61E+03	5.61E+04	2.57E+05	3.62E+01	1.76E-01	4.36E-02	1.39E-02	1.06E-02
4.5	1.20E+02	9.49E+02	4.00E+04	2.99E+05	1.23E+02	8.02E-02	2.39E-02	1.50E-02	1.38E-02
5	1.86E+02	2.95E+03	4.90E+04	3.95E+05	2.86E+01	6.44E-02	4.36E-02	1.85E-02	1.60E-02

Table S3- $\hat{r}$ MSE, T=10

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	4.61E+01	7.39E+01	1.17E+02	3.37E+02	2.40E+02	4.61E+02	5.19E+03	5.88E+02	5.23E+02
2	7.80E+01	2.38E+02	3.47E+02	7.90E+03	1.92E+04	2.96E+04	1.43E+04	7.69E+01	1.41E+00
2.5	1.88E+02	2.10E+02	3.01E+03	3.43E+04	8.62E+04	6.66E+04	7.02E+00	4.81E-02	1.28E-01
3	1.11E+02	2.98E+02	1.68E+04	5.93E+04	2.08E+05	1.86E+04	4.58E-02	2.63E-01	1.55E-02

3.5	1.69E+02	7.70E+02	2.79E+04	1.04E+05	2.37E+04	2.12E-01	1.20E+01	3.18E-02	1.09E-02
4	6.21E+03	1.73E+03	5.82E+04	2.76E+05	3.69E+01	1.88E-01	4.43E-02	1.39E-02	1.10E-02
4.5	1.33E+02	1.05E+03	4.19E+04	3.28E+05	1.25E+02	8.68E-02	2.41E-02	1.52E-02	1.39E-02
5	2.00E+02	3.13E+03	5.17E+04	4.45E+05	2.90E+01	6.94E-02	4.37E-02	1.85E-02	1.63E-02

Table S4- $\hat{r}$ bias, T=20

r_0 \ %	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	2.32E+00	2.69E+00	3.44E+00	4.27E+00	5.87E+00	6.35E+00	6.91E+00	6.44E+00	5.51E+00
2	2.55E+00	3.83E+00	5.84E+00	7.67E+00	1.50E+01	1.95E+01	2.56E+00	1.39E-01	1.44E-03
2.5	2.20E+00	4.53E+00	7.10E+00	1.29E+01	2.93E+01	1.09E+00	1.83E-02	1.34E-02	4.52E-03
3	2.74E+00	5.15E+00	9.98E+00	2.27E+01	2.07E+01	3.69E-02	2.33E-02	1.04E-02	9.29E-03
3.5	2.32E+00	5.56E+00	1.03E+01	5.21E+01	1.61E-01	5.95E-02	1.26E-02	8.72E-03	4.70E-03
4	2.12E+00	6.33E+00	2.08E+01	9.40E+01	2.64E-02	5.52E-02	1.11E-02	5.45E-03	1.29E-02
4.5	2.68E+00	6.57E+00	2.27E+01	8.26E+01	5.15E-02	4.17E-02	5.36E-03	6.07E-03	7.07E-03
5	1.67E+00	6.18E+00	2.51E+01	6.12E+01	7.94E-02	2.51E-02	2.41E-03	8.09E-03	1.05E-02

Table S5- $\hat{r}$ variance, T=20

r_0 \ %	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	2.67E+01	2.29E+01	3.12E+01	4.94E+01	1.55E+02	1.16E+02	2.15E+02	2.33E+02	3.42E+02
2	3.36E+01	4.84E+01	9.24E+01	1.58E+02	5.09E+03	8.35E+03	3.59E+02	2.96E+00	9.24E-03
2.5	2.51E+01	6.80E+01	1.47E+02	2.66E+03	1.90E+04	1.23E+02	2.82E-02	7.99E-03	3.87E-03
3	5.76E+01	9.52E+01	1.38E+03	9.60E+03	1.86E+04	7.12E-02	1.16E-02	5.63E-03	3.23E-03
3.5	4.84E+01	1.26E+02	7.84E+02	6.08E+04	6.93E+00	5.17E-02	8.06E-03	6.53E-03	4.34E-03
4	5.14E+01	3.63E+02	1.07E+04	1.08E+05	3.83E-01	4.64E-02	1.04E-02	5.83E-03	5.20E-03
4.5	1.62E+02	4.39E+02	1.34E+04	7.89E+04	2.67E-01	2.74E-02	1.06E-02	6.53E-03	6.34E-03
5	5.98E+01	2.06E+02	1.65E+04	6.34E+04	2.21E-01	2.09E-02	1.19E-02	7.66E-03	8.33E-03

Table S6- r MSE, T=20

r_0 \ %	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	3.21E+01	3.01E+01	4.31E+01	6.76E+01	1.89E+02	1.57E+02	2.63E+02	2.75E+02	3.72E+02
2	4.01E+01	6.31E+01	1.27E+02	2.16E+02	5.32E+03	8.73E+03	3.66E+02	2.98E+00	9.24E-03
2.5	2.99E+01	8.85E+01	1.97E+02	2.83E+03	1.99E+04	1.24E+02	2.86E-02	8.17E-03	3.89E-03
3	6.51E+01	1.22E+02	1.48E+03	1.01E+04	1.90E+04	7.26E-02	1.22E-02	5.74E-03	3.31E-03
3.5	5.37E+01	1.57E+02	8.89E+02	6.35E+04	6.95E+00	5.53E-02	8.22E-03	6.61E-03	4.36E-03
4	5.59E+01	4.03E+02	1.11E+04	1.17E+05	3.83E-01	4.94E-02	1.05E-02	5.86E-03	5.36E-03
4.5	1.70E+02	4.82E+02	1.39E+04	8.58E+04	2.70E-01	2.92E-02	1.07E-02	6.57E-03	6.39E-03
5	6.26E+01	2.44E+02	1.71E+04	6.72E+04	2.27E-01	2.15E-02	1.19E-02	7.72E-03	8.44E-03

Table S7- $\hat{r}$ bias, T=50

r_0 \ %	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	1.40E+00	1.84E+00	2.35E+00	2.82E+00	3.07E+00	3.35E+00	2.85E+00	1.92E+00	9.06E-01
2	1.61E+00	2.53E+00	3.76E+00	4.43E+00	7.60E+00	3.91E+00	5.69E-02	2.81E-03	-7.53E-04
2.5	1.52E+00	3.15E+00	4.95E+00	7.82E+00	4.96E+00	1.32E-01	7.77E-03	-2.20E-03	-1.06E-03
3	1.37E+00	3.48E+00	7.14E+00	1.17E+01	9.50E-01	1.03E-02	8.36E-03	-5.11E-03	3.07E-04
3.5	1.14E+00	3.53E+00	6.85E+00	1.13E+01	-2.53E-02	1.99E-02	2.63E-03	1.71E-03	-1.41E-03
4	8.30E-01	3.49E+00	7.42E+00	1.31E+01	-1.74E-02	1.37E-02	-3.57E-03	-4.10E-03	1.70E-03
4.5	4.98E-01	3.35E+00	1.03E+01	1.09E+01	1.31E-02	1.14E-02	-5.22E-03	-2.19E-03	-1.31E-03

5	1.02E-01	3.41E+00	1.30E+01	1.03E+01	3.46E-02	-4.45E-04	-5.69E-03	-2.19E-03	-3.58E-04

Table S8- $\hat{r}$ variance, T=50

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	7.38E+00	1.03E+01	1.44E+01	3.05E+01	2.91E+01	4.99E+01	3.90E+01	3.08E+01	1.48E+01
2	1.10E+01	1.87E+01	3.85E+01	5.60E+01	2.33E+03	4.47E+02	2.29E-01	1.36E-02	3.04E-03
2.5	1.72E+01	3.38E+01	7.91E+01	3.33E+02	2.48E+02	6.47E+00	9.98E-03	2.72E-03	1.46E-03
3	1.68E+01	4.37E+01	6.67E+02	2.36E+03	4.11E+01	2.40E-02	4.00E-03	1.97E-03	1.44E-03
3.5	2.07E+01	4.95E+01	1.63E+02	8.44E+02	1.35E-01	1.64E-02	2.94E-03	2.23E-03	1.78E-03
4	2.08E+01	5.73E+01	2.51E+02	3.54E+03	9.16E-02	1.07E-02	3.94E-03	2.15E-03	2.01E-03
4.5	2.16E+01	6.14E+01	1.24E+03	2.86E+03	8.20E-02	7.25E-03	4.31E-03	2.55E-03	2.27E-03
5	2.24E+01	8.67E+01	3.02E+03	4.90E+03	7.54E-02	6.54E-03	4.90E-03	3.54E-03	3.35E-03

Table S9- $\hat{r}$ MSE, T=50

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	9.33E+00	1.37E+01	1.99E+01	3.84E+01	3.86E+01	6.11E+01	4.72E+01	3.44E+01	1.57E+01
2	1.36E+01	2.51E+01	5.27E+01	7.57E+01	2.39E+03	4.62E+02	2.33E-01	1.36E-02	3.04E-03
2.5	1.95E+01	4.37E+01	1.04E+02	3.94E+02	2.73E+02	6.49E+00	1.00E-02	2.73E-03	1.46E-03
3	1.87E+01	5.58E+01	7.18E+02	2.50E+03	4.20E+01	2.41E-02	4.07E-03	2.00E-03	1.44E-03
3.5	2.21E+01	6.20E+01	2.10E+02	9.72E+02	1.36E-01	1.68E-02	2.95E-03	2.23E-03	1.78E-03
4	2.14E+01	6.94E+01	3.06E+02	3.71E+03	9.19E-02	1.09E-02	3.96E-03	2.17E-03	2.02E-03
4.5	2.18E+01	7.26E+01	1.35E+03	2.98E+03	8.22E-02	7.38E-03	4.34E-03	2.56E-03	2.27E-03
5	2.24E+01	9.83E+01	3.19E+03	5.00E+03	7.65E-02	6.54E-03	4.93E-03	3.55E-03	3.35E-03

Table S10- $\hat{\gamma}$ bias, T=10

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	2.50E+02	1.74E+02	4.24E+02	2.49E+02	6.48E+01	9.46E+01	1.10E+01	5.61E+00	2.98E+00
2	2.61E+02	1.10E+02	5.75E+01	1.65E+01	1.39E+01	2.22E+00	1.13E+01	1.79E-01	3.19E-02
2.5	1.21E+02	1.19E+02	9.92E+00	7.28E+00	1.92E+00	3.63E-01	2.81E-02	-3.95E-02	-2.07E-02
3	3.10E+02	2.36E+02	7.04E+01	3.14E+00	1.29E+00	1.28E-02	-4.43E-02	6.85E-02	-3.42E-03
3.5	3.90E+01	1.87E+02	6.00E+00	2.31E+00	1.55E-01	-1.89E-02	-4.62E-02	-1.56E-02	1.69E-02
4	3.97E+02	1.10E+02	1.16E+01	2.21E+00	1.16E-01	-4.09E-02	-1.63E-02	2.29E-03	3.44E-03
4.5	4.99E+01	1.57E+02	4.23E+00	1.06E+01	6.44E-02	-3.70E-02	2.56E-03	1.05E-02	1.05E-02
5	1.29E+02	1.01E+01	3.77E+00	6.62E-01	3.31E-02	-3.32E-02	1.22E-03	1.62E-02	-1.12E-02

Table S11- $\hat{\gamma}$ variance, T=10

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	2.74E+06	1.87E+06	1.47E+07	7.35E+06	1.87E+05	8.67E+05	8.83E+03	1.56E+03	3.09E+02
2	5.95E+06	1.30E+06	1.46E+05	1.15E+04	1.98E+04	6.94E+01	2.29E+04	1.85E+00	4.18E-01
2.5	6.60E+05	1.16E+06	1.64E+03	1.81E+03	3.42E+01	3.03E+00	2.73E-01	1.40E-01	1.15E-01
3	9.91E+06	6.93E+06	8.42E+05	8.02E+01	4.20E+01	1.64E-01	8.06E-02	1.33E+00	4.50E-02
3.5	3.96E+04	3.88E+06	7.95E+02	4.74E+01	4.09E-01	9.60E-02	5.93E-02	5.14E-02	2.74E-02
4	1.30E+07	2.68E+06	1.24E+04	4.53E+01	1.58E-01	6.44E-02	3.05E-02	1.40E-02	4.14E-02
4.5	1.02E+05	2.07E+06	1.29E+02	2.22E+04	1.01E-01	3.88E-02	1.37E-02	1.36E-02	2.78E-02
5	2.66E+06	1.32E+03	8.52E+01	3.66E+00	8.23E-02	2.43E-02	1.34E-02	3.63E-02	1.25E-02

Table S12- $\hat{\gamma}$ MSE, T=10

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	2.80E+06	1.90E+06	1.48E+07	7.41E+06	1.92E+05	8.76E+05	8.95E+03	1.59E+03	3.17E+02
2	6.02E+06	1.31E+06	1.50E+05	1.17E+04	2.00E+04	7.44E+01	2.30E+04	1.88E+00	4.19E-01
2.5	6.74E+05	1.17E+06	1.74E+03	1.87E+03	3.79E+01	3.16E+00	2.74E-01	1.42E-01	1.16E-01
3	1.00E+07	6.98E+06	8.47E+05	9.00E+01	4.37E+01	1.64E-01	8.25E-02	1.33E+00	4.50E-02
3.5	4.11E+04	3.92E+06	8.31E+02	5.28E+01	4.33E-01	9.64E-02	6.14E-02	5.17E-02	2.77E-02
4	1.31E+07	2.69E+06	1.25E+04	5.02E+01	1.71E-01	6.61E-02	3.07E-02	1.41E-02	4.14E-02
4.5	1.04E+05	2.09E+06	1.47E+02	2.24E+04	1.05E-01	4.02E-02	1.37E-02	1.37E-02	2.79E-02
5	2.68E+06	1.42E+03	9.94E+01	4.10E+00	8.34E-02	2.54E-02	1.34E-02	3.66E-02	1.26E-02

Table S13- $\hat{\gamma}$ bias, T=20

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	3.75E+01	2.31E+01	4.51E+01	6.41E+01	8.74E+00	2.24E+00	1.63E+00	1.08E+00	7.60E-01
2	2.90E+02	1.08E+01	1.43E+01	2.39E+00	1.52E+00	7.76E-01	4.00E-01	7.16E-02	3.41E-02
2.5	4.37E+01	9.09E+00	2.74E+00	1.69E+00	7.73E-01	1.41E-01	8.25E-03	-2.17E-02	9.41E-03
3	2.37E+01	5.29E+00	2.39E+00	1.27E+00	3.12E-01	1.20E-02	-1.67E-02	-5.94E-03	-8.43E-03
3.5	8.20E+01	1.72E+01	2.12E+00	9.23E-01	9.72E-02	-2.11E-02	-1.19E-02	-2.17E-03	1.22E-02
4	1.10E+01	8.01E+00	1.88E+00	6.58E-01	5.26E-02	-3.66E-02	-1.96E-03	-8.16E-03	-5.24E-03
4.5	1.51E+01	3.11E+00	1.77E+00	4.70E-01	2.44E-02	-2.12E-02	4.22E-03	5.05E-03	3.33E-03
5	7.34E+00	2.98E+00	1.65E+00	2.49E-01	4.19E-03	-1.11E-02	4.64E-03	6.66E-03	-7.09E-03

Table S14- $\hat{\gamma}$ variance, T=20

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	3.84E+04	2.25E+04	2.59E+05	4.75E+05	1.02E+04	4.65E+01	3.20E+01	1.87E+01	1.40E+01
2	2.29E+07	5.90E+03	3.54E+04	2.98E+01	1.53E+01	8.26E+00	2.34E+00	4.12E-01	1.32E-01
2.5	2.42E+05	8.00E+03	3.09E+01	1.42E+01	6.19E+00	4.81E-01	8.29E-02	5.04E-02	2.76E-02
3	4.82E+04	7.61E+02	2.29E+01	8.81E+00	1.82E+00	6.07E-02	2.89E-02	1.82E-02	1.13E-02
3.5	1.67E+06	5.54E+04	1.72E+01	5.56E+00	1.02E-01	3.84E-02	1.09E-02	8.62E-03	9.99E-03
4	7.06E+03	6.41E+03	1.32E+01	3.20E+00	5.37E-02	2.52E-02	6.98E-03	5.33E-03	7.25E-03
4.5	2.46E+04	3.26E+01	1.14E+01	2.09E+00	3.70E-02	1.48E-02	4.67E-03	5.25E-03	5.69E-03
5	9.94E+02	2.88E+01	9.95E+00	6.04E-01	2.80E-02	8.39E-03	3.33E-03	4.82E-03	4.54E-03

Table S15- $\hat{\gamma}$ MSE, T=20

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	3.98E+04	2.30E+04	2.62E+05	4.79E+05	1.03E+04	5.15E+01	3.46E+01	1.98E+01	1.46E+01
2	2.30E+07	6.02E+03	3.56E+04	3.55E+01	1.76E+01	8.86E+00	2.50E+00	4.17E-01	1.33E-01
2.5	2.44E+05	8.08E+03	3.84E+01	1.71E+01	6.79E+00	5.01E-01	8.29E-02	5.08E-02	2.77E-02
3	4.87E+04	7.89E+02	2.87E+01	1.04E+01	1.92E+00	6.08E-02	2.91E-02	1.83E-02	1.14E-02
3.5	1.67E+06	5.57E+04	2.17E+01	6.41E+00	1.12E-01	3.88E-02	1.11E-02	8.63E-03	1.01E-02
4	7.18E+03	6.47E+03	1.67E+01	3.63E+00	5.65E-02	2.65E-02	6.98E-03	5.40E-03	7.27E-03
4.5	2.49E+04	4.23E+01	1.45E+01	2.31E+00	3.76E-02	1.52E-02	4.69E-03	5.27E-03	5.70E-03
5	1.05E+03	3.77E+01	1.27E+01	6.66E-01	2.81E-02	8.52E-03	3.35E-03	4.87E-03	4.59E-03

Table S16- $\hat{\gamma}$ bias, T=50

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	4.52E+00	3.17E+00	2.24E+00	1.49E+00	9.70E-01	6.01E-01	3.07E-01	1.26E-01	4.71E-02

2	3.36E+00	2.20E+00	1.35E+00	8.21E-01	3.87E-01	9.86E-02	7.49E-02	1.22E-02	9.12E-03
2.5	2.94E+00	1.74E+00	1.08E+00	5.29E-01	1.03E-01	4.59E-02	-1.97E-03	8.90E-04	3.59E-03
3	2.71E+00	1.58E+00	9.07E-01	3.34E-01	7.42E-02	4.08E-03	-1.01E-02	6.80E-03	-1.37E-03
3.5	2.55E+00	1.49E+00	7.99E-01	1.91E-01	3.73E-02	-1.08E-02	-5.19E-03	-1.99E-03	6.66E-03
4	2.44E+00	1.41E+00	7.14E-01	1.09E-01	2.48E-02	-1.31E-02	-1.15E-04	-9.92E-04	-3.39E-03
4.5	2.34E+00	1.35E+00	6.44E-01	8.62E-02	8.21E-03	-8.21E-03	2.36E-03	1.01E-04	1.66E-03
5	2.25E+00	1.30E+00	5.91E-01	4.54E-02	-3.13E-03	-3.34E-03	1.57E-03	3.42E-03	-1.16E-03

Table S17- $\hat{\gamma}$ variance, T=50

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	6.97E+01	4.58E+01	2.63E+01	1.36E+01	1.02E+01	8.16E+00	6.42E+00	4.70E+00	3.27E+00
2	4.52E+01	1.97E+01	8.36E+00	5.58E+00	3.49E+00	1.79E+00	3.75E-01	9.78E-02	4.62E-02
2.5	2.80E+01	9.13E+00	5.51E+00	3.03E+00	1.17E+00	9.36E-02	3.10E-02	1.58E-02	1.02E-02
3	2.32E+01	7.47E+00	4.03E+00	1.75E+00	2.06E-01	2.23E-02	1.02E-02	5.15E-03	4.36E-03
3.5	2.03E+01	6.40E+00	3.11E+00	9.59E-01	2.84E-02	1.46E-02	3.22E-03	3.44E-03	3.09E-03
4	1.78E+01	5.70E+00	2.55E+00	5.01E-01	1.89E-02	7.87E-03	2.42E-03	2.00E-03	2.34E-03
4.5	1.48E+01	5.20E+00	2.15E+00	2.45E-01	1.43E-02	3.92E-03	1.63E-03	1.73E-03	1.85E-03
5	1.19E+01	4.82E+00	1.83E+00	9.00E-02	1.14E-02	2.44E-03	1.23E-03	1.71E-03	1.24E-03

Table S18- $\hat{\gamma}$ MSE, T=50

r_0 \ γ_0	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	9.02E+01	5.59E+01	3.14E+01	1.58E+01	1.11E+01	8.52E+00	6.51E+00	4.72E+00	3.28E+00
2	5.65E+01	2.45E+01	1.02E+01	6.25E+00	3.64E+00	1.80E+00	3.81E-01	9.80E-02	4.63E-02
2.5	3.67E+01	1.22E+01	6.67E+00	3.31E+00	1.18E+00	9.57E-02	3.10E-02	1.58E-02	1.02E-02
3	3.05E+01	9.97E+00	4.85E+00	1.86E+00	2.11E-01	2.23E-02	1.03E-02	5.19E-03	4.36E-03
3.5	2.68E+01	8.61E+00	3.75E+00	9.96E-01	2.97E-02	1.48E-02	3.25E-03	3.45E-03	3.13E-03
4	2.37E+01	7.69E+00	3.06E+00	5.13E-01	1.95E-02	8.04E-03	2.42E-03	2.00E-03	2.36E-03
4.5	2.03E+01	7.02E+00	2.57E+00	2.52E-01	1.44E-02	3.99E-03	1.63E-03	1.73E-03	1.85E-03
5	1.69E+01	6.52E+00	2.17E+00	9.21E-02	1.14E-02	2.45E-03	1.23E-03	1.72E-03	1.24E-03

Table S19. Bias, variance, and MSE of parameter estimates for $\hat{r}$ and $\hat{\gamma}$ from simulations with increased noise ($\sigma_0 = 0.25$), stationary (S) and perturbed (P) samples. Triplets indicate values for time series of length T at 10, 20, and 50. We have put the stationary $\sigma = 0.05$ for ease of comparison.

Parameter: $\hat{r}$	Bias	Variance	MSE
Data			
S, $\sigma_0 = 0.05$, $\gamma_0 = 1$	3.94, 2.74, 1.37	95.61, 57.63, 16.79	111.11, 65.11, 18.66
S, $\sigma_0 = 0.05$, $\gamma_0 = 5$	0.02, 0.01, 3e-4	0.02, 0.01, 1e-3	0.02, 3e-3, 1e-3
S, $\sigma_0 = 0.25$, $\gamma_0 = 1$	31e2, 24e2, 15e2	27e7, 24e7, 34e6	2.8e8, 2.5e8, 3.6e7
S, $\sigma_0 = 0.25$, $\gamma_0 = 5$	0.03, 0.01, 8e-4	0.24, 0.09, 0.03	0.24, 0.09, 0.03
P, $\sigma_0 = 0.05$, $\gamma_0 = 1$	0.12, 0.09, 0.09	0.38, 0.32, 0.27	0.39, 0.33, 0.28
P, $\sigma_0 = 0.05$, $\gamma_0 = 5$	0.01, 3e-4, 1e-4	0.01, 3e-3, 1e-3	0.01, 3.2e-3, 1.2e-3
Parameter: $\hat{\gamma}$			
S, $\sigma_0 = 0.05$, $\gamma_0 = 1$	3.10e3, 23.74, 2.71	99e5, 4.14e4, 23.18	10e6, 4.87e4, 30.50

S, $\sigma_0 = 0.05,\ \gamma_0 = 5$	-3e-3, -0.01, -1e-3	0.04, 0.01, 4e-3	0.04, 0.01, 4e-3
S, $\sigma_0 = 0.25,\ \gamma_0 = 1$	164.26, 8.45, 1.35	19e5, 66e2, 303.69	2.0e6, 6.7e3, 305.52
S, $\sigma_0 = 0.25,\ \gamma_0 = 5$	0.71, 0.02, -0.01	0.61, 0.18, 0.07	0.62, 0.18, 0.07
P, $\sigma_0 = 0.05,\ \gamma_0 = 1$	0.05, 0.04, 0.04	0.05, 0.04, 0.03	0.05, 0.04, 0.03
P, $\sigma_0 = 0.05,\ \gamma_0 = 5$	2e-3, -1e-3, -3e-3	0.04, 0.01, 4e-3	0.04, 0.01, 4.5e-3

Table S20- ML estimate variance-covariance matrix from the time series in Fig. 4a-b, which uses $r_0 = 3,\ \gamma_0 = 5,\ K_0 = (r_0 - 1)^{1/\gamma_0},\ \sigma_0 = 0.05$, and $T = 50$.

	$\hat{r}$	$\hat{K}$	$\hat{\gamma}$	$\hat{\sigma}$
$\hat{r}$	1.19e-3	-1.70e-4	-1.06e-3	3.80e-8
$\hat{K}$	-1.70e-4	4.70e-5	3.92e-4	-1.08e-e8
$\hat{\gamma}$	-1.06e-3	3.92e-4	4.01e-3	-9.61e-8
$\hat{\sigma}$	3.80e-8	-1.08e-8	-9.61e-8	2.22e-e5

Table S21- ML estimate variance-covariance matrix from the time series in Fig. 4c-d, which uses $r_0 = 3,\ \gamma_0 = 1,\ K_0 = (r_0 - 1)^{1/\gamma_0},\ \sigma_0 = 0.05$, and $T = 50$.

	$\hat{r}$	$\hat{K}$	$\hat{\gamma}$	$\hat{\sigma}$
$\hat{r}$	11.8	-5.63	-5.86	1.09e-4
$\hat{K}$	-5.63	2.69	2.80	-5.18e-5
$\hat{\gamma}$	-5.86	2.80	2.97	-5.40e-5
$\hat{\sigma}$	1.09e-4	-5.18e-5	-5.40e-5	2.16e-5

Figure Captions

Figure S1- Per capita growth rate as a function of density for several different values of the shape parameter γ and arbitrary r.

Figure S2- r-γ pairs used in simulation (points) and the bifurcation boundary (line) from damped to sustained oscillations.

Figure S1

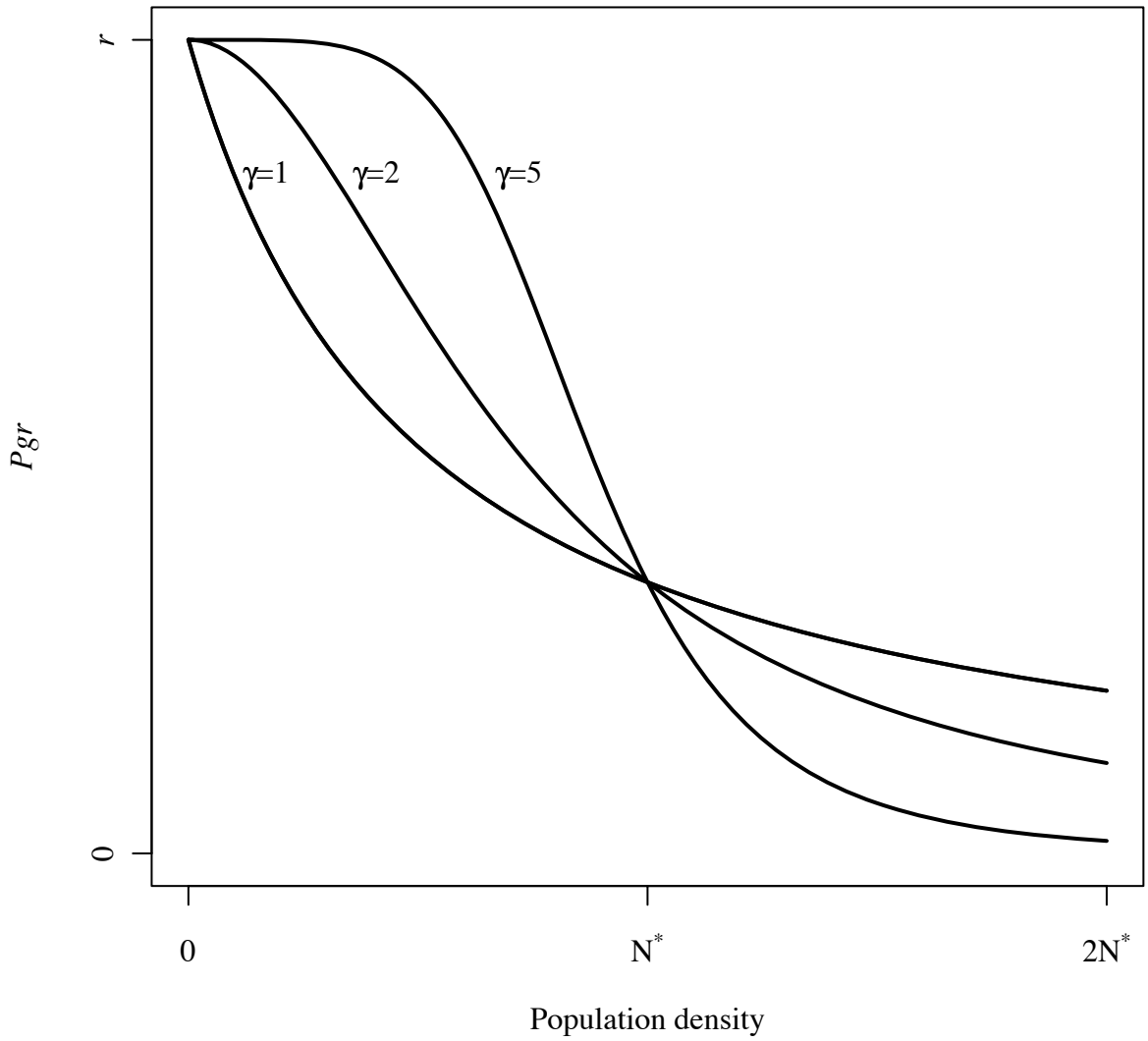
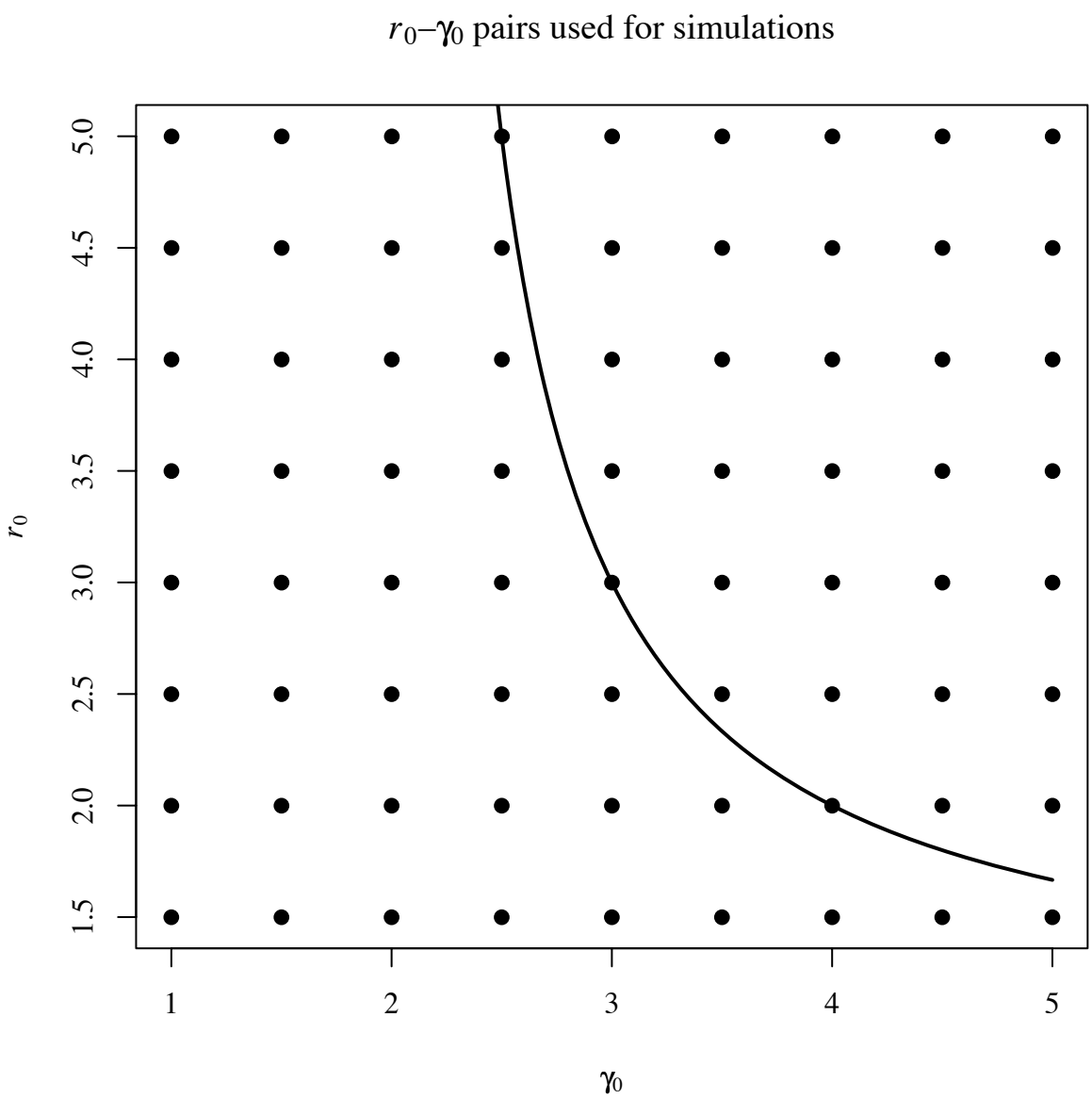


Figure S2


r_0–γ_0 pairs used for simulations
r_0
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1
2
3
4
5
γ_0